Aliasing of a time series shifts high-frequency variance into lower frequencies. It is caused by sampling at an interval too broad to resolve true high-frequency signals. This effect is well understood in classical time series analysis, where sample intervals are known and constant, but in a geologic context, where sample intervals may vary, aliasing remains poorly known. We address here the aliasing problem relevant to the search for orbital variations recorded in sedimentary sections. Our example is aliasing of the 23,000-year period orbital precession rhythm which is common in late Quaternary paleoclimatic records. We illustrate three cases of aliasing. First, we sample precession at a constant interval of 25,000 years. This is a typical target for many "high-resolution" studies of Neogene sections. This sampling interval translates precessional variance into predictable spectral peaks near 400 ka and 100 ka, which could be mistaken for the longer-period eccentricity rhythms. Second, random variations in the sampling interval around the 25,000-year target spread the aliased variance over a range of frequencies. This induces either unpredictable long-period spectral peaks or, in the extreme, a white noise spectrum. In the third case, variations of the sampling interval are autocorrelated. This simulates a section with long-period variations in sedimentation rate sampled at constant depth intervals. Here the single aliased peaks of case 1 are split into two or more peaks of slightly higher and lower frequencies. In all three cases, for long enough time series, the total variance recorded is the same. We compare these numerical experiments to a Miocene oxygen isotope record from Deep Sea Drilling Project site 577, sampled at ~25,000-year intervals. With these data it is impossible to tell whether the long-period variations are due to the direct effects of eccentricity or the aliased effects of precession. In theory it should be possible to test for eccentricity signals at low resolution by randomizing the sampling intervals. In practice, however, it is only through high-density sampling that we can define intervals well enough to assess the effects of aliasing.

INTRODUCTION

Detailed time series and statistical analyses of late Pleistocene climate records have demonstrated that variations in the Earth's orbital parameters, eccentricity of the orbit and tilt and precession of the rotation axis, all play an important role in controlling variations in global climate [Imbrie et al., 1984; Martinson et al., 1987]. These discoveries have prompted researchers interested in pre-Pleistocene climates to search for evidence of Milankovitch-related frequencies in sedimentary records of oceanic and climatic change.

This note is not to dispute the hypothesis that changes in the Earth's orbital configuration may play an important role in controlling climatic variations during the geologic past, but rather to emphasize the need for adequate sampling strategies to detect Milankovitch frequencies in the geologic record. Specifically, our concern is that some studies of
The problem of aliasing is not a new one. It is discussed in many texts describing techniques of time series analysis and is well understood in fields utilizing signal processing such as seismology [Bendat and Piersol, 1971; Jenkins and Watts, 1968]. The problem of aliasing, which is associated with geologic sections, where variation in the time domain sampling interval is present, is not well described in texts discussing time series analyses. Here we discuss aliasing relevant to the study of climate change in geologic sections and demonstrate the need for an adequate sampling strategy to determine the true nature of climatic variability in the geologic record. Only in this way can we properly test the hypothesis that orbital changes play an important role in controlling climate in the remote geologic past.

### Simple Cases of Aliasing

Aliasing of a time series results from sampling at an interval (Δt) that is too large to adequately resolve the true nature of a periodic signal. The highest frequency (shortest period) that can be resolved in a data set, the Nyquist frequency, is equal to 1/[2*Δt] if the number of data points available is even and is equal to (N-1)/[2N*Δt] if the number of data points, N, is odd. For example, if a geologic time series is sampled at 5000-year intervals and the number of data points is even, then the shortest period that could be resolved is 10,000 years.

The Nyquist frequency is more properly called the "Nyquist folding frequency." This is because the variance of a frequency component higher than the Nyquist frequency is "folded" into the spectrum at lower frequencies. For example, in Figure 1a, a hypothetical geologic "data series" is plotted. The period of cyclic variations in this series is equal to 1.3 m in this section. If we sample at 1.5-m intervals, the sampled record has an apparent (aliased) period of nearly 8 m. All frequencies contained in the true time series that are higher than the Nyquist frequency appear in the "observed" time series as frequency components in the interval from zero to the Nyquist frequency.

An example of aliasing of a real data set is shown in Figure 1b. Here the calcium carbonate content from a section of Deep Sea Drilling Project (DSDP) site 572 from the equatorial Pacific is plotted. The low-resolution samples (labeled with solid triangles) were analyzed on the drill ship, while the higher-resolution samples (connected by solid curve) were measured as part of a shore-based study [Pisias and Prell, 1985]. If the shipboard data were the only analyses available, we would conclude that the carbonate content of the sediments was essentially constant. The more detailed data, however, show clearly that a regular pattern of variation in carbonate content is present in this equatorial Pacific site. This example is an extreme case where the aliased data not only failed to show the true frequency of variations but also were unable to reconstruct the true range of values in this sediment record.

In general, the nature of aliasing can be predicted if we know the sampling interval and the types of frequencies likely to be present in the real world. Specifically, if f is an observed frequency in the range between zero and the Nyquist frequency f_{Nyq}, spectral variance at f could result from true variance at f or from frequencies (f_{aliased}) higher than f_{Nyq} and equal to:

\[ f_{aliased} = (2f_{Nyq} \pm f), (4f_{Nyq} \pm f), \ldots, (2nf_{Nyq} \pm f) \]

(where n is any integer). To illustrate the folding of a true frequency into the frequency range zero to f_{Nyq}, consider the time series of orbital precession shown in Figure 2. The time series of precession was calculated from the coefficients of Berger [1977] using a sample interval of 5000 years (Figure 2a). This sampling is adequate to resolve the dominant periods of precession at 23,000 and 19,000 years. The time series was calculated for 1 million years.
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The variance spectrum for the time series sampled at 5000-year intervals is shown in Figure 2b. The Nyquist frequency is equivalent to a period of 10,000 years, and the spectrum contains the dominant spectral peaks at 23,000 and 19,000 years.

The time series obtained from sampling precession at 25,000-year intervals is shown by the circles in Figure 2a and is reproduced in Figure 2c. The aliased spectrum of this time series is shown in Figure 2d, where the Nyquist frequency is now 50,000 years and the dominant spectral peaks of 23,000 and 19,000 years are aliased to spectral peaks of 454,000, 217,000, and 80,000 years. The 80,000-year peak is an alias of the 19,000-year peak, while the 454,000- and 217,000-year peaks are aliases of the first two primary components of precession of 23,700 and 22,400-year peaks, respectively [Berger, 1977].

How does this aliased orbital record compare to geologic records with similar resolution? The aliased precession time series generated above and calculated over a 2-m.y. period is compared to an isotopic record from DSDP site 588 published by Hodell et al. [1986] (Figure 3). The average sampling interval of this paleoclimate record is 25,000 years. Figure 3 shows both the raw data and data smoothed with a five-point running average. These data have been analyzed recently for their frequency content by Tiwari [1987], who concluded that long-period Milankovitch cyclicity was present. The similarity of the aliased precession curve and the isotopic data is striking, and we suggest that there is a strong possibility of aliasing in the site 588 data. Tiwari [1987] used Walsh spectral analysis to examine cyclicity in the $\delta^{18}O$ data from site 588 and showed a dominance of low-frequency "power" with spectral peaks at a number of frequencies. A number of the periods, 93,000, 227,000, and 455,000 years, are very close to the aliased periods we predict for precession sampled at 25,000-year intervals (80,000, 217,000, and 454,000 years). Tiwari [1987] did not address the possibility of sample aliasing; we argue that aliasing cannot be excluded. If this is true, then the conclusions of Tiwari [1987] about the presence of Milankovitch frequencies related directly to eccentricity are incorrect.

Aliasing with variable Sample Intervals

Thus far, we have only discussed the theoretical effect of aliasing for a time series which has been sampled at a known constant time interval. In general, sediment accumulation rates vary through time, but sediment sections are sampled at constant depth sampling intervals. Thus, time sampling intervals may be variable in geologic time series even if the time scale is known perfectly. What is the effect on the observed time series, especially on aliasing and primary frequency composition, in the...
case where the sampling interval is not constant as assumed? The question of sedimentation rate changes has in part been addressed by Herterich and Sarthein [1984] and Martinson et al., [1982] who discussed the effects of time scale errors on the nature of the frequency spectrum. These studies, however, did not address the question of aliasing, which is the question addressed here.

To answer this question, we performed two sets of numerical experiments to gain insights about how variations in the sampling interval of a geologic sequence affect the frequency domain and time domain statistics of an observed time series. In these experiments we examine the effect of only one source of error in the study of a geologic time series. We assume that the overall chronology of the time series is well known, and we investigate the effect of having the sampling interval not constant. In the first set of experiments we assumed that the variations in the sampling interval are truly random and independent. In the second set of experiments we used the time domain statistics of variations in sedimentation rates inferred from late Pleistocene sediment cores to model more geologically reasonable variations in sampling intervals. For these cores, variations in sampling interval were calculated using age estimates obtained from an orbitally tuned time scale [Imbrie et al., 1984]. Again, we emphasize that we do not address problems caused by errors in the time scale assigned to a geologic record. In addition, how these errors and natural variations in sampling affect the coherence between time series is not discussed. All of these problems are important, but our intent here is to focus on the problems associated with sampling the geologic record at a resolution too low to fully define the true nature of oceanographic and climatic change.

In Figure 4 we compare the calculated precession of the Earth sampled at constant 25,000-year intervals with the same signal sampled with randomly distributed variations in the sample interval. As discussed above, the Nyquist frequency is 1/50 kyr, so true variance at higher frequencies is aliased to periods longer than 50 kyr. The effects of these different degrees of variability in the true sample interval on the variance spectra are shown in Figure 5. For the case where the true sampling interval varies randomly (Figures 5b-5d), the aliased peaks of precession appear in each numerical simulation except when the maximum level of variation (with the sampling interval ranging from 1 to 49 kyr) was used. The variance explained by each of these peaks is very different from the case with no sample interval variations (Figure 5a). Note that the scaling on the variance axes in Figure 5 changes by 1 order of magnitude from the case where the sampling interval is constant to the case where the variations in sample interval lengths is maximum. Random variations in the length of the sampling interval cause the variance associated with each spectral (aliased) peak to be distributed over a wider frequency band than in the case of constant sample intervals. Thus the aliased peaks become smaller in magnitude. For the case of completely random length in sampling intervals (Figure 5d), the aliased variance is spread over a very wide range of frequencies, and identification of the original peaks is not possible.

Variations in the sampling interval of geologic time
Fig. 4. (a) Precession sampled at 5000-year intervals. (b) Precession sampled at 25,000-year intervals. (c) Realization of precession sampled at a mean sampling interval of 25,000 years with randomly distributed variations ranging from 10,000 to 40,000 years. (d) Realization of precession sampled with autoregressive estimated variation in the sampling intervals. The mean interval is 25,000 years, ranging from 10,000 to 40,000 years.

series are not truly random, but rather more systematic, as the major source of variation would be produced by changes in sedimentation rates. To examine this possibility, we used the time scale estimated for core V28-238 by Imbrie et al. [1984], generated under the hypothesis of orbitally forced climate change. Using the variations in sedimentation rates implied by the orbital time scale developed for this core, we calculated a series of realistic sample intervals in two ways. If core V28-238 were sampled at constant depth intervals of 40 cm, then the corresponding time intervals would average 25 kyr, but they would range from 16 to 30 kyr. If instead we take every fifth sample analyzed by Shackleton and Opdyke [1973], the spacing between samples ranges from 23 to 54 cm, or in time units from 12 to 39 kyr. This second time series of sampling intervals was modeled using a second-order autoregressive model [Box and Jenkins, 1970]. This autoregressive model was then used to generate a sample series with the same statistical properties as the sample interval variations for core V28-238. The orbital precession record was then aliased by sampling at these variable intervals over a 5-million-year length. The intervals were scaled to have a mean and standard deviations of 25 ± 3 (range 17-33), 25 ± 5 (range 10-40), and 25 ± 8 kyr (range 1-49 kyr) (Figures 5f-5h). By comparison with V28-238, the most geologically reasonable case is Figure 5f, with sample interval ranges from 10 to 40 kyr.

As in the case of truly random variations in length of the sampling interval, the second set of experiments reduced the amount of variance in the observed "aliased" spectral peaks relative to the constant interval case. Unlike the random case, however, the changes in the spectra are more systematic. In the autoregressive experiments, the aliased spectral peaks are split into peaks of slightly higher and lower frequencies than the expected aliased peak. This is because the sampling interval variations reflect long wavelength changes in sedimentation rates. The time sampling intervals are, in general, longer in sections of lower sedimentation rates and shorter in sections of high sedimentation rates. In the high sedimentation rate intervals the aliased signal will appear to have a long period (lower frequency), while in the low sedimentation rate intervals the aliased signal will have a slightly shorter period (higher frequency). As the total variance is not significantly changed by the sample interval variations (that is, the total integrated area of the variance spectra must remain the same), the split peaks have lower
Fig. 5. Effect on aliased variance spectrum of different amounts and types of variation in sample intervals (Δt) around a mean of 25 kyr. (a) Constant Δt of 25 kyr, with time series shown in Figure 4b. (b) Random variations in Δt from 17 to 33 kyr. (c) Random variations in Δt from 10 to 40 kyr, with time series shown in Figure 4c. (d) Random variations from 1 to 49 kyr. (e) Constant Δt of 25 kyr (same as Figure 5a). (f) Autoregressive variations in Δt from 17 to 33 kyr. (g) Autoregressive variations in Δt from 10 to 40 kyr, with time series shown in Figure 4d. (h) Autoregressive variations from 1 to 49 kyr.
amplitudes than the original aliased peaks (Figure 5).

If we apply the same sets of sample interval variation to the time series of eccentricity, which has a 100,000-year wavelength (lower frequency than the Nyquist frequency), the variations in sampling interval have little effect on the observed variance spectra. However, the 41,000-year period obliquity signal is of higher frequency than the Nyquist frequency and, like the precession frequencies, would also be subject to aliasing.

CONCLUSIONS

These experiments place us in an interesting quandary. If we have an accurate time scale and thus a known sampling interval, then the effect of aliasing may be predictable. Randomization of the sampling interval reduces the amount of variance that aliasing may be predictable. Randomization of the sampling interval reduces the amount of variance that aliasing will insert into discrete frequencies of the spectrum by distributing the aliased variance over a wider-frequency band. These variations do not make significant changes in the spectra of nonaliased longer-period components. Thus, a sufficiently random sampling interval, perhaps induced by long-period variations in the sedimentation rate, might reduce the aliasing problem. However, to answer the question as to whether the spectra for the site 588 data reflect true eccentricity influence or aliased precessional influence requires us to know the ratio of variance of the long-period eccentricity cycle relative to that of the precession cycle. If precession-related variance is relatively small, then inherent variations in sampling intervals will help reveal the true low-frequency spectra even at sampling intervals of 25,000 years. However, if precession accounts for a large amount of the observed variation, then the variations in sampling interval will not help. In this case, reducing the variance in individual aliased peaks may not be enough to reveal true long-period components.

Unfortunately, the only way to determine the relative amounts of true long-period variance, versus variance contributed by aliasing, is by sufficiently close sampling of the original geologic section so that the effect of aliasing can be fully defined.

If aliasing has occurred, this raises the possibility that the long-period changes observed in site 588 in fact reflect variations associated with higher frequencies of precession. If so, then Tiwari's [1987] conclusion that orbital variations are important in controlling climate during this interval is correct, but the suggestion that the source of the variations is the Earth's eccentricity cycle is incorrect. The low-resolution sampling does not allow us to adequately determine the true source of these variations, changes in the distribution of solar insolation due to the precession of the Earth's orbit or changes in total insolation resulting from changes in the eccentricity of the orbit. Only with proper high-resolution sampling will be able to reliably tell the difference between long-period climate variations and an aliased record containing the shorter-period fluctuations related to the Earth's orbital parameters.

REFERENCES


